Stability behavior of second order neutral impulsive stochastic differential equations with delay

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Abstract: In this article, we study the existence and asymptotic stability in pth moment of mild solutions to second order neutral stochastic partial differential equations with delay. Our method of investigating the stability of solutions is based on fixed point theorem and Lipschitz conditions being imposed.

Keywords: Stochastic, neutral, impulse, asymptotic stability, mild solution

1. INTRODUCTION

Stochastic partial differential equations have received much attention in many areas of science including physics, biology, medicine and engineering. The existence, uniqueness and asymptotic behavior of solutions of the first order stochastic partial differential equations have been considered by several authors [3, 4, 11, 12, 15, 26]. Moreover many dynamical systems not only depend on present and past states but also involve derivatives with delays. Deterministic neutral functional differential equations, which was originally introduced by Hale and Lunel [9], are of great interest in theoretical and practical applications. Kolmanovskii and Myshkis [13] introduced neutral stochastic functional differential equations and gave its applications in chemical engineering and aero elasticity considering environmental disturbances into account.

Caraballo et al. [5] have considered the exponential stability of neutral stochastic delay partial differential equations by the Lyapunov functional approach. In [7], Dauer and Mahmudov have analyzed the existence of mild solutions to semilinear neutral evolutions with nonlocal conditions by using the fractional power of operators and Kransnoselski-Schaefer type fixed point theorem. In [10], Hu and Ren have established the existence results for impulsive neutral stochastic functional integrodifferential equations with infinite delays. It is well known that classical technique applied in the study of stability is based on a stochastic version of the Lyapunov direct method. However the Lyapunov direct method has some difficulty with the theory and application to specific problems when discussing the asymptotic behaviour of solutions in stochastic differential equations [16]. It seems that new methods are required to address those difficulties.

Appleby [1] studied the almost sure stability of stochastic differential equations with fixed point approach. Luo [18, 17] have successfully applied fixed point principle to investigate the stability of mild solutions of stochastic equations. Luo and Taniguchi [19], have studied the asymptotic stability of neutral stochastic partial differential equations with infinite delay by using the fixed point theorem. The impulsive effects exist widely in many evolution processes in which states are changed abruptly of certain moments of time, involving such fields as finance, economics, mechanics, electronics and telecommunications, etc [25]. The theory of impulsive differential equations have been studied extensively in 21, 22. However in addition to impulsive effects, stochastic effects likewise exist in real systems. It is well known that a lot of dynamical systems have variable structures subjects to stochastic abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, etc.

Even though there are many valuable results about neutral stochastic partial differential equations, they are mainly concerned with first-order case. In many cases it is advantageous to treat the second order stochastic differential equations rather than to convert them to first-order systems. The second-order stochastic differential equations are the right model in continuous time to account for integrated processes than can be made stationary. For instance, it is useful for engineers to model mechanical vibrations or charge on a capacitor or condenser subjected to white noise excitation through a second-order stochastic differential equations. The studies of the qualitative properties about abstract deterministic second order evolution equation governed by the generator of a strongly continuous cosine family was proposed in 8, 27. Mahmudov and McKibben [20] established results concerning the global existence and approximate controllability of mild solutions for a class of second order stochastic evolution equations. Moreover, Ren [2] and Sun [23] established the existence, uniqueness and stability of the second-order neutral impulsive stochastic evolution equations with delay with some non-Lipschitz conditions. Balasubramaniam and Muthukumar [2] also discussed the approximate controllability of second-order neutral stochastic distributed implicit functional differential equations with infinite delay. Sakhivel et al. [24] have studied the asymptotic stability of second-order neutral stochastic distributed implicit functional differential equations with infinite delay. Let Zhang et al. [14] have studied the controllability of second-order semilinear impulsive stochastic neutral functional evolution equations. Inspired by this consideration, the main objective of this paper is to study the asymptotic stability of the second-order neutral impulsive stochastic delay differential equations.

2. PRELIMINARIES

In this section, we briefly give some basic definitions and results for stochastic equations in infinite dimensions and cosine families of operators. We refer to Prato and Zabczyk [6] and Fattorini [8] for more details. Let \( X \) and \( E \) be two real separable Hilbert spaces and \( L(E, X) \) be the space of bounded linear operators from \( E \) into \( X \), equipped with the usual operator norm \( \| \cdot \| \). Let \( (\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P}) \) be a complete probability space furnished with a normal filtration \( \{\mathcal{F}_t\}_{t \geq 0} \) generated by the smooth process \( w \).
on \((\Omega, \Gamma, P)\) with the linear bounded covariance operator \(Q\) such that \(trQ < \infty\). We assume that there exist a complete orthonormal system \(\{e_i\}_{i \geq 1}\) in \(E\), a bounded sequence of nonnegative real numbers \(\{\lambda_i\}\) such that \(Qe_i = \lambda_i e_i, i = 1, 2, 3, \ldots\), and a sequence \(\{\beta_i\}\) of independent Brownian motions such that
\[
\langle w(t), e \rangle = \sum_{i=1}^{\infty} \sqrt{\lambda_i} \langle e_i, e \rangle \beta_i(t), \ e \in E
\]
and \(\Gamma_i = \Gamma_i^{w}\), where \(\Gamma_i^{w}\) is the sigma algebra generated by \(\{w(s); t \geq 0\}\). Let \(L_2^{0} = L_2(Q^{1/2}E; X)\) denote the space of all Hilbert–Schmidt operators from \(Q^{1/2}E\) to \(X\) with the inner product \(\langle \psi, \varphi \rangle_{L_2^{r}} = tr[\psi \varphi^*]\). Let \(L'(Q, \Gamma, X)\) be the Hilbert space of all \(\Gamma\)-measurable square integrable random variables with values in a Hilbert space \(X\).

In this paper, we consider the following second-order neutral impulsive stochastic differential equations with delays of the form
\[
d(t)(x(t) - f_i(t, x(t - \sigma(t)))) = [Ax(t) + f_1(t, x(t - \rho(t)))]dt + f_2(t, x(t - \sigma(t)))d\omega(t), \quad t \geq 0, (1)
\]
\[
\varphi_0(\cdot) = \varphi \in D_{\Gamma_0}^{\beta}, \quad x'(0) = x_i, (2)
\]
\[
\Delta x(t_k) = I_k(x(t_k^+)) \Delta x'(t_k) = \tilde{I}_k(x(t_k)), (3)
\]
where \(\varphi \in D_{\Gamma_0}^{\beta}\) and \(x_i\) is also an \(\Gamma_0\)-measurable \(X\)-valued random variables independent of \(w\).

\(A : D(A) \subset X \to X\) is the infinitesimal generator of a strongly continuous cosine family on \(X\);
\(f_i : \mathbb{R} \times X \to X\) for \(i = 0, 1, 2\); let \(f_2 : \mathbb{R} \times X \to L_2^{0}\) are appropriate mappings and \(I_k, \tilde{I}_k \) are appropriate functions. Moreover, let
\[
t_0 = t_0 < t_1 < \ldots < t_m < t_{m+1} = \infty,
\]
\[
\Delta x(t_k) = x(t_k^+) - x(t_k^-), \quad \Delta x'(t_k) = x'(t_k^+) - x'(t_k^-), (4)
\]
de note the right and left limits of \(x\) at \(t_k\). Similarly \(x'(t_k^+), x'(t_k^-)\) denote the right and left limits of \(x'\) at \(t_k\).

Moreover \(I_k, \tilde{I}_k\) represents the size of the jump. Let \(\delta, \rho, \sigma : \mathbb{R} \to [0, \tau] (\tau > 0)\) are continuous. The space \(D\) is assumed to be equipped with the norm
\[
\|x\|_D = \sup_{-\tau \leq s \leq 0} \|x(s)\|_X.
\]
Here \(D_{\Gamma_0}^{\beta}([-\tau, 0]; X)\) denote the family of all almost surely bounded, \(\Gamma_0\)-measurable, continuous random variables from \([-\tau, 0]\) to \(X\).

Let us introduce the spaces
\[
H([0, T]; X) = \{x : J \to X, x_{[t_k, t_{k+1}]} \in C((t_k, t_{k+1}], X), \ k = 1, 2, 3, \ldots, m \}
\]
and there exist \(x(t_k^+), x(t_k^-)\) for \(k = 1, 2, 3, \ldots, m\) and
\[
H'([0, T]; X) = \{x \in H([0, T]; X), x_{[t_k, t_{k+1}]} \in C'((t_k, t_{k+1}], X), \ k = 1, 2, 3, \ldots, m \}
\]
and there exist \(x'(t_k^+), x'(t_k^-)\) for \(k = 1, 2, 3, \ldots, m\).

It is obvious that \(H([0, T]; X)\) and \(H'([0, T]; X)\) are Banach spaces endowed with the norm
\[
\|x\|_{H'} = \sup_{t \in [0, T]} \|x(t)\|_X.
\]

In this section, we mention some basic concepts, notations, and properties about cosine families of operators \([8, 27]\).

Let \(L(E; X)\) is the space of bounded linear operators from \(E\) into \(X\). The one parameter family
\[
\{C(t); t \in R\} \subset L(X)
\]
satisfying
(i) \(C(0) = I\),
(ii) \(C(t) x\) is continuous in \(t\) on \(R\) for all \(x \in X\),
(iii) \(C(t + s) - C(t - s) = 2C(t)C(s)\)

for all \(t, s \in R\) is called a strongly continuous cosine family.

The corresponding strongly continuous sine family
\[
\{S(t); t \in R\} \subset L(X)
\]
is define
\[
S(t)x = \int_{0}^{t} C(s)xds, \ t \in R, x \in X.
\]

The generator \(A : X \to X\) of \(\{C(t); t \in R\}\) is given by
\[
Ax = \left(\frac{d^2}{dt^2} + C(t)x\right)_{t=0}
\]
for all \(x \in D(A) = \{x \in X : C(.)x \in C^2(R; X)\}\)

It is well known that the infinitesimal generator \(A\) is a closed, densely defined operator on \(X\). Such cosine and sine families and their generators satisfy the following properties.

**Lemma 2.1:** \([8]\)

Suppose that \(A\) is the infinitesimal generator of a cosine family of operators \(\{C(t); t \in R\}\). Then the following terms hold.
3. ASYMPTOTIC STABILITY OF SECOND-ORDER NEUTRAL STOCHASTIC DIFFERENTIAL EQUATIONS

Now let us present the main result of this paper. We consider the asymptotic stability in the pth moment of mild solutions (1), (2), (3) by using the fixed point principle. Moreover, for the purpose of asymptotic stability, we shall assume that in this work \( f_1(t, 0) = 0 \) \((i = 0, 1)\) and \( f_2(t, 0) = 0 \). \( I_k(0) = 0, \tilde{I}_k(0) = 0, k = 1, 2, ..., m \).

Then equations (1), (2) and (3) have a trivial solution when \( \phi \equiv 0 \) and \( x_0 \equiv 0 \).

To prove the following result, we impose the following conditions.

(i) The cosine family of operators \( \{C(t); t \geq 0\} \) on \( X \) and the corresponding sine family \( \{S(t); t \geq 0\} \) satisfy the conditions

\[ \left\| C(t) \right\|_X \leq Me^{-\alpha t}, \quad \left\| S(t) \right\|_X \leq Me^{-\beta t}, \quad t \geq 0 \]

for some constants \( M, \alpha, \beta \geq 0 \).

(ii) The functions \( f_1(i = 0, 1, 2) \) satisfy the Lipschitz condition and there exist positive constants \( K_1, K_2, K_3 \) for every \( t \geq 0 \) and \( x, y \in X \), such that

\[ \left\| f_1(t, x) - f_1(t, y) \right\|_X \leq K_1 \| x - y \|_X; \quad i = 0, 1, \]

\[ \left\| f_2(t, x) - f_2(t, y) \right\|_X \leq K_2 \| x - y \|_X; \quad i = 2 \]

(iii) The function \( I_k, \tilde{I}_k \) and there are positive constants \( q_k, g_k \) such that

\[ \left\| I_k(x) - I_k(y) \right\|_X \leq q_k \| x - y \|_X, \]

\[ \left\| \tilde{I}_k(x) - \tilde{I}_k(y) \right\|_X \leq g_k \| x - y \|_X, \]

for each \( x, y \in X, k = 1, 2, 3, ..., m \).

(iv) \( I_k(0) = 0, \tilde{I}_k(0) = 0, k = 1, 2, 3, ..., m \).

**Theorem 3.1:**

Assume the conditions (i)-(iv) hold. Let \( p \geq 2 \) be an integer and \( c_p = \left( \frac{p(p - 1)}{2} \right)^{\frac{p}{2}} \). If the inequality

\[ \left[ S^p M^p \left( k_1 b^p + k_2 a^p + k_3 2a \right) + \hat{L} + \hat{D} \right] < 1 \]

is satisfied, then the second-order neutral stochastic differential equations with delays (1), (2) and (3) is asymptotically stable in pth moment.

**Proof:** Define an operator \( \Psi : H \rightarrow H \) by
\[
\Psi(x)(t) = C(t)\varphi(0) + S(t)\left(x_1 - f_0(0, x(0, -\delta(0)))\right) + \int_0^t C(t - s)f_0(s, x(s, \delta(s)))ds + \int_0^t S(t - s)f_1(s, x(s, -\rho(s)))ds + \int_0^t S(t - s)f_2(s, x(s, -\sigma(s)))dw(s) + \sum_{0 \leq t_k < t} C(t - t_k)I_k(x(t_k)) + \sum_{0 \leq t_k < t} S(t - t_k)\tilde{I}_k(x(t_k)) = \sum_{i=1}^7 F_i(t), \quad t \geq 0 .
\]

(5)

In order to prove the asymptotic stability, it is enough to prove that the operator \( \Psi \) has a fixed point \( H \). To prove this result, we use the contraction mapping principle. To apply the contraction mapping principle, first we verify the mean square continuity of \( \Psi \) on \( [0, \infty) \).

Let \( x \in H \), \( t_i \geq 0 \) and \( \| r \| \) is sufficiently small then

\[
E\left\| \Psi(x)(t_i + r) - \Psi(x)(t_i) \right\|^p \leq 7^{p-1} \sum_{i=1}^7 E\left\| F_i(t_i + r) - F_i(t_i) \right\|^p .
\]

We can see that

\[
E\left\| F_i(t_i + r) - F_i(t_i) \right\|^p \to 0, \quad i = 1, 2, 3, 4, 6, 7 \text{ as } r \to 0 .
\]

Moreover by using Holder’s inequality and lemma 2.2, we obtain

\[
E\left\| F_5(t_i + r) - F_5(t_i) \right\|^p \leq 2^{p-1} c_p \int_0^{t_i + r} \left\| S(t_i + r - s) - S(t_i - s) \right\|^{(2/p)} ds 
\]

\[
+ 2^{p-1} c_p \int_0^{t_i + r} \left\| S(t_i + r - s)f_2(s, x(s, -\sigma(s))) \right\|^{(2/p)} ds 
\]

\[
\to 0 \text{ as } r \to 0 ,
\]

(6)

Thus, \( \Psi \) is continuous in \( p \)th moment on \( [0, \infty) \).

Next, we show that \( \Psi(H) \subseteq H \). From (5), we obtain

\[
E\left\| \psi x(t) \right\|^p \leq 8^{p-1} E\left\| C(t)\varphi(0) \right\|^p + 8^{p-1} E\left\| S(t)x \right\|^p + 8^{p-1} E\left\| S(t)f_0(0, x(0, -\delta(0))) \right\|^p
\]

\[
+ 8^{p-1} E\left\| C(t - s)f_0(s, x(s, -\delta(s)))ds \right\|^p + 8^{p-1} E\left\| S(t - s)f_1(s, x(s, -\rho(s)))ds \right\|^p
\]

\[
+ 8^{p-1} E\left\| S(t - s)f_2(s, x(s, -\sigma(s)))dw(s) \right\|^p + 8^{p-1} \sum_{0 \leq t_k < t} E\left\| C(t - t_k)I_k(x(t_k)) \right\|^p
\]

\[
+ 8^{p-1} \sum_{0 \leq t_k < t} E\left\| S(t - t_k)\tilde{I}_k(x(t_k)) \right\|^p.
\]

Now, we estimate the terms on the right hand side of (7) using (I), (II), (III) and (IV) we obtain

\[
8^{p-1} E\left\| C(t)\varphi(0) \right\|^p \leq 8^{p-1} M^p e^{-bt} \left\| \varphi \right\|^p \to 0 \text{ as } t \to \infty ,
\]

(8)

\[
8^{p-1} E\left\| S(t)x \right\|^p \leq 8^{p-1} M^p e^{-ap} \left\| x \right\|^p \to 0 \text{ as } t \to \infty ,
\]

(9)

\[
8^{p-1} E\left\| S(t)f_0(0, x(0, -\delta(0))) \right\|^p \leq 8^{p-1} M^p e^{-ap} K_1 \left\| x(0, -\delta(0)) \right\|^p \to 0 \text{ as } t \to \infty ,
\]

(10)

\[
8^{p-1} \sum_{0 \leq t_k < t} E\left\| C(t - t_k)I_k(x(t_k)) \right\|^p \leq 8^{p-1} M^p e^{-bt} \left\| I_k(x(t_k)) \right\|^p \to 0 \text{ as } t \to \infty ,
\]

(11)

\[
8^{p-1} \sum_{0 \leq t_k < t} E\left\| S(t - t_k)\tilde{I}_k(x(t_k)) \right\|^p \leq 8^{p-1} M^p e^{-bt} \left\| \tilde{I}_k(x(t_k)) \right\|^p \to 0 \text{ as } t \to \infty ,
\]

(12)

From I, II, III, (IV) and Holder’s inequality, we have

\[
8^{p-1} E\left\| C(t - s)f_0(s, x(s, -\delta(s)))ds \right\|^p \leq 8^{p-1} M^p K_1 \left\| e^{-b(t-s)} \right\| \int_0^{t-s} e^{-b(t-s)} E\left\| x(s, -\delta(s)) \right\|^p ds
\]

\[
\leq 8^{p-1} M^p K_1 b^p \int_0^{t-s} e^{-b(t-s)} E\left\| x(s, -\delta(s)) \right\|^p ds.
\]

(13)
For any $x(t) \in H$ and any $\varepsilon > 0$, there exist a $t_1 > 0$, such that $E \| x(s - \delta(s)) \|_X^p < \varepsilon$ for $t \geq t_1$.

Thus from (13), we obtain

$$8^{p-1} E \left[ \int_0^t C(t-s) f_0(s,x(s-\delta(s))) ds \right]^p_X \leq 8^{p-1} M^p K^p \int_0^t e^{-bt} E \| x(s-\delta(s)) \|_X^p ds$$

+ $8^{p-1} M^p K^p b^{-p} \varepsilon$. (14)

As $e^{-bt} \to 0$ as $t \to \infty$ and by assumption on Theorem 3.1, there exists a $t_2 > t_1$, such that for any $t \geq t_2$, we have

$$8^{p-1} M^p K^p b^{-p} e^{-bt} \int_0^t e^{bt} E \| x(s-\delta(s)) \|_X^p ds$$

$$\leq \varepsilon - 8^{p-1} M^p K^p b^{-p} \varepsilon \quad \text{(15)}$$

From (14) and (15), for any $t \geq t_2$, we obtain

$$8^{p-1} E \left[ \int_0^t C(t-s) f_0(s,x(s-\delta(s))) ds \right]^p_X < \varepsilon.$$ (16)

That is to say,

$$8^{p-1} E \left[ \int_0^t C(t-s) f_0(s,x(s-\delta(s))) ds \right]^p_X \to 0$$

as $t \to \infty$. (16)

Similarly we can obtain

$$8^{p-1} E \left[ \int_0^t S(t-s) f_1(s,x(s-\rho(s))) ds \right]^p_X \to 0$$

as $t \to \infty$. (17)

Now for any $x(t) \in S$, $t \in [-\tau, \infty]$, we have

$$8^{p-1} E \left[ \int_0^t S(t-s) f_1(s,x(s-\rho(s))) ds \right]^p_X \to 0$$

Thus from (8)-(12) and (16), (17), (21), we can obtain

$$E \| \Psi_x(t) \|_X^p \to 0 \text{ as } t \to \infty.$$ (21)

Next we prove that $\Psi$ is a contraction mapping. To see this, let $x, y \in H$ and for $s \in [0,T]$, we obtain

$$\sup_{s \in [0,T]} E \left[ \| \Psi_x(t) - \Psi_y(t) \|_X^p \right]$$

$$\leq 8^{p-1} c_p M^p k^p \left[ \int_0^t e^{-2a(t-s)} (E \| x(s-\sigma(s)) \|_X^p)^{2/p} ds \right]$$

+ $8^{p-1} c_p M^p k^p \left[ \int_0^t e^{-2a(t-s)} (E \| x(s-\rho(s)) \|_X^p)^{2/p} ds \right]$. (18)

As $e^{-pat} \to 0$ as $t \to \infty$ and by assumption on Theorem 3.1, there exists a $t_2 > t_1$, such that for any $t \geq t_2$, we have

$$8^{p-1} c_p M^p k^p \left[ \int_0^t e^{-2a(t-s)} (E \| x(s-\sigma(s)) \|_X^p)^{2/p} ds \right]$$

$$\leq \varepsilon - 8^{p-1} c_p M^p k^p \varepsilon (2a)^{-p/2} \quad \text{(19)}$$

From (18) and (19), for any $t \geq t_2$, we obtain

$$8^{p-1} E \left[ \int_0^t S(t-s) f_2(s,x(s-\sigma(s))) ds \right]^p_X < \varepsilon. \quad \text{(20)}$$

That is to say

$$8^{p-1} E \left[ \int_0^t S(t-s) f_2(s,x(s-\sigma(s))) ds \right]^p_X \to 0 \text{ as } t \to \infty.$$ (21)
\[ + 5^{p-1} \sup_{s \in [0,T]} \mathbb{E} \left[ \int_{0}^{\tilde{T}} \left( f_{2}(s, x(s - \sigma(s))) - f_{2}(s, y(s - \sigma(s))) \right) d\omega(s) \right]_{Y}^{p} \]

\[ + 5^{p-1} \sup_{s \in [0,T]} \mathbb{E} \left[ \sum_{k \in \mathcal{I}} \left( I_{k} (x(t_{k})) - I_{k} (y(t_{k})) \right) \right]_{Y}^{p} \]

\[ + 5^{p-1} \sup_{s \in [0,T]} \mathbb{E} \left[ \sum_{k \in \mathcal{I}} S(t - t_{k}) \left( \tilde{I}_{k} (x(t_{k})) - \tilde{I}_{k} (y(t_{k})) \right) \right]_{Y}^{p} \]

\[ \leq 5^{p-1} M^{p} \left( k_{b}^{p} b^{-p} + k_{a}^{p} a^{-p} + k_{c}^{p} c_{p} (2a)^{-p/2} \right) \]

\[ + e^{-pbT} \mathbb{E} \left[ \sum_{k=1}^{m} \left\| \eta_{k} \right\|_{X}^{p} \right] + e^{-paT} \mathbb{E} \left[ \sum_{k=1}^{m} \left\| \xi_{k} \right\|_{X}^{p} \right] \]

\[ \times \sup_{s \in [0,T]} \mathbb{E} \left[ \left\| x(t) - y(t) \right\|_{X}^{p} \right]. \]

Therefore, \( \Psi \) is a contraction mapping and hence there exist an unique fixed point \( x(\cdot) \) in \( H \) which is the solution of the equations (1)-(3) with \( x_{0}(\cdot) = \varphi, x'(0) = x \) and \( \mathbb{E} \left[ \left\| x(t) \right\|_{X}^{p} \right] \to 0 \) as \( t \to \infty \). This completes the proof.

**Corollary 3.2:**
If the conditions (I) to (IV) hold, then the second-order neutral impulsive stochastic differential system (1) to (3) is mean square asymptotically stable if

\[ 5M^{2} (k_{b}^{2} b^{-2} + k_{a}^{2} a^{-2} + k_{c}^{2} (2a)^{-p/2} + \hat{L} + \hat{D}) < 1 \]

where \( \hat{L} = e^{-2bT} \mathbb{E} \left[ \sum_{k=1}^{m} \left\| \eta_{k} \right\|_{X}^{p} \right] \), \( \hat{D} = e^{-2aT} \mathbb{E} \left[ \sum_{k=1}^{m} \left\| \xi_{k} \right\|_{X}^{p} \right] \).

**4. REFERENCES**


[19] Luo, J., Taniguchi, T., Fixed points and stability of stochastic neutral partial differential equations with


